A rock dropped from a cliff falls the last 1/3 of the cliff in 1 second. How high is the cliff?

$g =$ 9.81 m/s^2

$t_{lower} =$ 1 s

Calculate the heights from the quadratic

$$x_{cliff} = \frac{g(t_{lower})^2}{2} \quad [15 \pm \sqrt{(15^2 - 9)}] = \frac{g(t_{lower})^2}{2} \quad 15 \pm \quad 14.69694$$

$$x_{cliff} = \frac{9.81}{2} \quad 15 \pm \quad 14.69694$$

Examine the top 2/3 of the cliff

$x_{cliff+} =$ 4.905 29.6969 = 145.7 m

$(2/3)x_{cliff+} =$ 97.11

Find $v_{2/3+}$

$v_{2/3+} = \sqrt{[(4/3)gx_{cliff+}]} =$ 43.65 m/s

Find $t_{first\ 2/3+}$

$t_{fall,2/3+} = v_{2/3+}/g =$ 4.45 sec

$(v_{2/3+})^2 =$ 1905.28 m^2/s^2

$(2/3)gx_{cliff+} =$ 952.64

$v_{final+} = \sqrt{[(v_{2/3+})^2 + (2/3)gx_{cliff+}]} =$ 53.46 m/s

Find $t_{last\ 1/3+}$

$$t_{last\ 1/3+} = \frac{-v_{2/3+} \pm \sqrt{[(v_{2/3+})^2 + (2/3)gx_{cliff}]}}{g}$$

$$t_{last\ 1/3+} = \frac{-43.6 \pm \sqrt{[1905.3 + 952.6]}}{9.81}$$

$$t_{last\ 1/3+} = \frac{-43.64 \pm 53.46}{9.81}$$

$$\mathbf{t_{last\ 1/3+} = \frac{9.81}{9.81} = 1.0000}$$

Confirmed! Yes, x = 145.7 m works!

Examine the bottom 1/3 of the cliff

$x_{cliff-} =$ 4.905 0.30306 = 1.487 m

$(2/3)x_{cliff-} =$ 0.99

Find $v_{2/3+-}$

$v_{2/3-} = \sqrt{[(4/3)gx_{cliff-}]} =$ 4.41 m/s

Find $t_{first\ 2/3-}$

$t_{fall,2/3-} = v_{2/3-}/g =$ 0.45 sec

$(v_{2/3-})^2 =$ 19.44 m^2/s^2

$(2/3)gx_{cliff-} =$ 9.72

$v_{final-} = \sqrt{[(v_{2/3-})^2 + (2/3)gx_{cliff-}]} =$ 5.40 m/s

Find $t_{last\ 2/3-}$

$$t_{last\ 1/3-} = \frac{-v_{2/3-} \pm \sqrt{[(v_{2/3-})^2 + (2/3)gx_{cliff-}]}}{g}$$

$$t_{last\ 1/3-} = \frac{-4.41 \pm \sqrt{[19.44 + 9.72]}}{9.81}$$

$$t_{last\ 1/3-} = \frac{-4.41 \pm 5.40}{9.81}$$

$$\mathbf{t_{last\ 1/3-} = \frac{-9.81}{9.81} = -1.0000}$$

Oops!! This is a non-physical result!
It's a negative time.

Shoot rock up from bottom of cliff, how high in 1 sec?

$x_{up} = v_{up}t - (1/2)gt^2 =$ 0.496

This should be 1/3 of the cliff:

$|3x_{up}| =$ 1.4865 **Yes! This is what the negative root gives!**

$x_{cliff-} =$ 1.4865

Cliff heights from quadratic

Velocity at 2/3 of the cliff height $(v_{2/3})2 = 2g(x_{2/3} - x_0)$

Time to achieve $v_{2/3}$ from x = 0 $v = v_0 + gt$

Speed at tbottom of the cliff from $v_{2/3}$ $(v_{final})2 = (v2/3)2+2g(x_{2/3} - x_0)$

Confirm $t_{last\ 1/3}$ is 1 second

Maybe the rock shot up from the bottom!

It shoots up half a meter